



## Plane-wave scattering from half-wave dipole arrays

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*Published in:*  
I E E E Transactions on Antennas and Propagation

*Publication date:*  
1970

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*  
Jensen, N. E. (1970). Plane-wave scattering from half-wave dipole arrays. *I E E E Transactions on Antennas and Propagation*, 18(6), 829-831.

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plotted. Parameters for this case are

$$\begin{aligned}\phi_r &= 0 \\ \phi_s &= 0 \\ \phi_t &= 0 \\ \theta_r &= 0 \\ \theta_s &= 0 \\ b/a &= 200 \\ b/\lambda &= 0.170\end{aligned}$$

where the geometry of Fig. 4 applies.

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## Plane-Wave Scattering from Half-Wave Dipole Arrays

**Abstract**—A matrix equation for determination of plane-wave scattering from arrays of thin short-circuited dipoles of lengths about half a wavelength is derived. Numerical and experimental results are presented for linear, circular, and concentric circular arrays.

Recently, Sledge [1] has investigated plane-wave scattering by linear arrays of thin center-loaded dipoles by solving integral equations for the induced currents. When the array consists of short-circuited thin dipoles of lengths about half a wavelength, an analysis based upon network theory may be applied. The purpose of this communication is to present theoretical and experimental results in such cases.

The theoretical model is shown in Fig. 1(a) and consists of an array of  $N$  short-circuited thin dipoles of lengths  $[L_1, L_2, \dots, L_N]$  illuminated by a plane electromagnetic field  $E^{\text{inc}}$  of wavelength  $\lambda$ . The dipoles are located parallel to the  $z$  axis with the terminals in the  $xy$  plane. The induced current distributions on the elements are assumed to be sinusoidal. When open circuited, the scattered field from a thin dipole is negligible when  $L \simeq \lambda/2$  [2], so the open-circuit induced voltages  $[V_1, V_2, \dots, V_N]$  in the elements are given by [3]

$$V_i = E^{\text{inc}} \cdot h_i^{\text{eff}}, \quad i = 1, 2, \dots, N \quad (1)$$

where  $h_i^{\text{eff}}$  is the effective height [4] of element  $i$  when isolated from the other elements of the array;  $E_i^{\text{inc}}$  is the incident field at

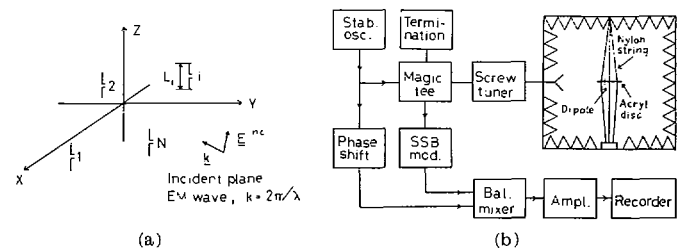


Fig. 1. (a) Theoretical model of dipole array. (b) Experimental setup.

element  $i$ . When shortcircuiting the dipoles, the terminal currents  $[I_1, I_2, \dots, I_N]$  follow from the  $N$ -terminal network theorem [3]

$$\begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad (2)$$

where  $Z_{ml}$  is the mutual impedance between elements  $m$  and  $l$ .

This equation is essentially identical with [1, eq. 10]. Similar equations have been derived by Richmond [5] for arrays of infinite long cylinders and Larsen [6] for interconnected array elements.

Assuming identical surroundings for all the elements, the self and mutual impedances are calculated on a digital computer from the formulas in [7] and [8] which apply well to the dipoles considered. The integrations in [8] are carried out numerically using Romberg's method [9]. Once the element currents are determined from (2), the reradiation pattern and backscattering cross section  $\sigma$  follow by standard techniques. Calculations show that the reciprocity theorem is fulfilled to a high degree of accuracy.

The experiments were carried out in a microwave anechoic chamber at X-band frequencies using the cancellation method, and the setup is shown schematically in Fig. 1(b). Using single sideband (SSB) modulation with a modulation frequency of 1 kHz, a frequency shift of 1 kHz is obtained in the backscattered signal which is linearly detected in a balanced mixer. The output should be proportional to the amplitude of the reflected signal and independent of its phase. Nonideal SSB modulation causes dependence on the phase, and each measurement consists of several recordings for various settings of the LO phase shifter. Generally phase sensitivities of  $\pm 0.2$  dB were observed.

The dipoles, made from 1-mm diameter brass, were mounted in a thin acrylic disk which was suspended in three 0.2-mm diameter nylon strings. Rotation and translation (used when measuring absolute values of  $\sigma$ ) were possible. The equipment proved capable of measuring  $\sigma$  of steel spheres down to a 5-mm diameter ( $\sigma/\lambda^2 \simeq 10^{-2}$ ) within  $\pm 0.5$  dB, and the presence of spheres down to a 3-mm diameter ( $\sigma/\lambda^2 \simeq 5 \cdot 10^{-4}$ ) could be detected.

Varying  $N$  and  $L$  for different linear, circular, and concentric circular array configurations, a large number of arrays with up to 26 elements have been investigated. Representative results are presented in Figs. 2 and 3. Fig. 2(a) shows the  $H$ -plane backscattering from a simple circular array of four  $\lambda/2$  dipoles and Fig. 2(b) from an array of two concentric circular arrays with dipoles of unequal lengths. In Fig. 3,  $H$ -plane backscattering patterns for linear arrays with four elements are shown. Comparing Figs. 3(a) and (b), one notes the Yagi-like enhancement of the backscattering from the array with different dipoles when looking toward the shorter dipoles. Similar effects were observed in concentric arrays where  $\sigma_{\text{max}}/\lambda^2$  for a  $2 \times 10$  elements array ( $ka_1$  and  $ka_2$  as in Fig. 2(b)) could be changed from 4.0 with equal length dipoles to 10.1, by shortening the outer elements to  $L/\lambda = 0.4$ .

The agreement between experiments and theory is seen to be very satisfactory, thus confirming the usefulness of this simple

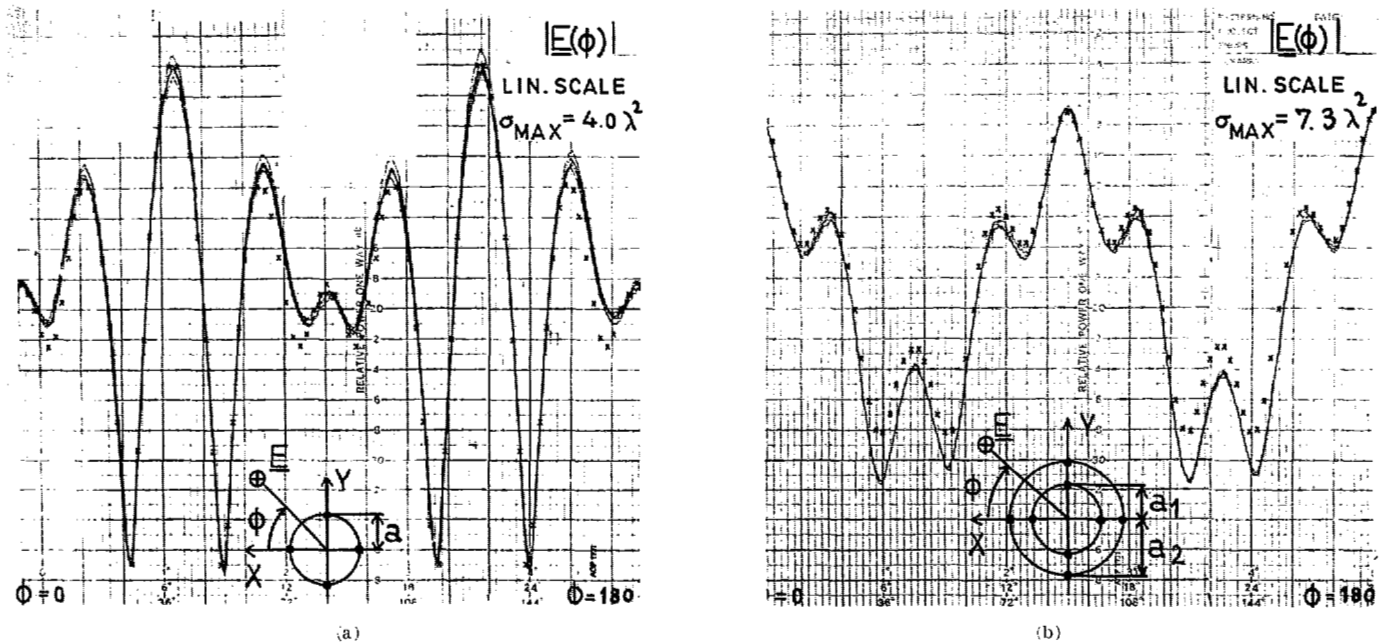


Fig. 2. Theoretical (x) and experimental H-plane backscattering patterns. (a) Circular array,  $ka = 4.0$ . Four elements of equal lengths,  $L = 0.50\lambda$ . (b) Concentric circular array,  $ka_1 = 2.04$ ,  $ka_2 = 4.08$ . Length of four outer elements is  $0.46\lambda$ , length of four inner elements is  $0.51\lambda$ .

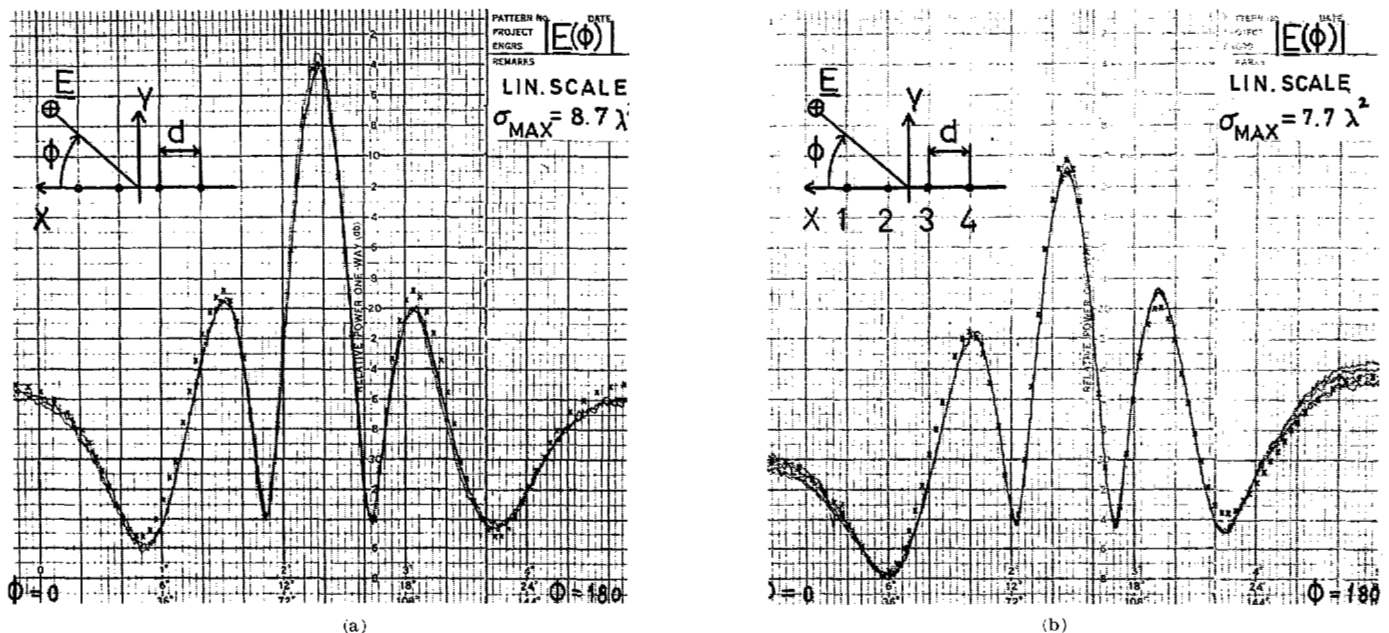


Fig. 3. Theoretical (x) and experimental H-plane backscattering patterns. (a) Linear array,  $kd = 2.04$ . Four elements of equal lengths,  $L = 0.51\lambda$ . (b) Linear array,  $kd = 2.04$ . Four elements of unequal lengths,  $L_1 = 0.61\lambda$ ,  $L_2 = 0.56\lambda$ ,  $L_3 = 0.51\lambda$ ,  $L_4 = 0.46\lambda$ .

theory of scattering from arrays of thin dipoles of lengths close to  $\lambda/2$ .

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### Effect of Electroacoustic Wave on the Radiation of a Plasma-Coated Spherical Antenna

**Abstract**—The radiation from a spherical antenna covered by a finite layer of lossy hot plasma is studied. When the plasma is cold, the antenna radiation can be recovered or enhanced when the plasma frequency is increased beyond the antenna frequency. When the plasma is hot, in addition to the phenomena for the cold plasma case, an electroacoustic wave may be excited in the plasma layer and lead to some resonances. The electroacoustic resonances may lead to a very strong antenna radiation. A good agreement was obtained between theory and experiment.

#### I. INTRODUCTION

Recently Messiaen and Vandenplas [1] studied the radiation of a spherical antenna covered by a layer of lossless cold plasma and observed that after the antenna radiation suffers cutoff, the antenna radiation can be recovered or enhanced when the plasma frequency exceeds the antenna frequency. Chen and Lin [2] observed experimentally a similar phenomenon of enhanced radiation from a cylindrical antenna covered by an overdense plasma. Lin and Chen [3] also investigated a spherical antenna covered by a layer of lossy cold plasma and obtained somewhat different results from that of Messiaen and Vandenplas. All these studies are, however, based on oversimplified models of plasma.

The present study was motivated by this new phenomenon of enhanced radiation. In the present study, a more realistic and complicated model is used. A spherical antenna is assumed to be covered by a finite layer of lossy hot plasma with an approximate plasma sheath existing between the antenna surface and the plasma layer. The antenna radiation is studied as a function of plasma parameters and antenna dimensions. It was found that the antenna radiation suffers the usual cutoff phenomenon, but it can be recovered or even greatly enhanced when the plasma frequency exceeds the antenna frequency. It was also found that the electroacoustic wave excited by the antenna in the plasma layer can cause some resonances and may lead to a strong antenna radiation. An experiment was conducted to confirm the theoretical results.

Manuscript received February 2, 1970; revised April 13, 1970. This work was supported by the NSF under Grant GK-2952.

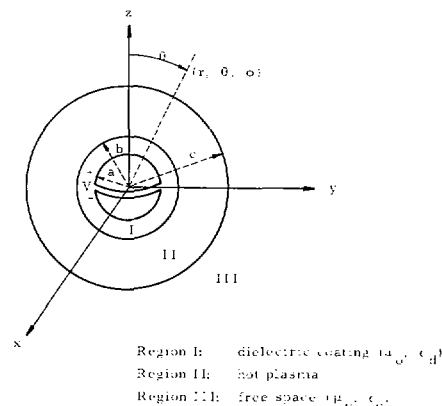


Fig. 1. A spherical antenna covered by lossy hot plasma layer.

The results obtained in this study are different from the work of Messiaen and Vandenplas [1] because the loss and temperature effects are included in the present study. The work of Wait [4] assumed a similar antenna model, but his antenna was imbedded in an infinite antenna so that entirely different results were obtained.

#### II. THEORY

The geometry of the problem is shown in Fig. 1. A spherical antenna of radius  $a$  is covered by a vacuum layer which approximates the plasma sheath. The outer surface of this vacuum sheath is at  $r = b$ . Over the vacuum sheath there is a spherical layer of uniform, lossy, and hot plasma with a thickness of  $(c - b)$ . Beyond this plasma layer is the free space. The spherical antenna is perfectly conducting except for a narrow equatorial gap between  $\pi/2 - \theta_1 \leq \theta \leq \pi/2 + \theta_1$ . Across this gap the antenna is driven by a constant voltage generator with a voltage of  $V$  and an angular frequency of  $\omega$ . The total space excluding the antenna is divided into three regions. Region I is the vacuum sheath, Region II is the lossy hot plasma layer, and the rest of the free space is Region III. The time dependence of  $\exp(j\omega t)$  is assumed in the analysis.

##### A. Fields in the Vacuum Sheath—Region I ( $a \leq r \leq b$ )

The plasma sheath formed on the antenna surface is approximated by a layer of vacuum sheath. Maxwell's equations in this region lead to an equation for the  $H_1$  field as

$$(\nabla^2 + \beta_0^2)H_1 = 0 \quad (1)$$

where  $\beta_0^2 = \omega^2\mu_0\epsilon_0$  and  $H_1$  is the magnetic field in Region I. With the rotational symmetry, (1) can be solved to give

$$H_{1\phi} = \frac{1}{r^{1/2}} \sum_{n=1}^{\infty} P_n^{1/2}(\cos \theta) [A_n H_{n+(1/2)}^{(2)}(\beta_0 r) + B_n H_{n+(1/2)}^{(1)}(\beta_0 r)]. \quad (2)$$

$H_{1r}$ ,  $H_{1\theta}$ , and  $E_{1\phi}$  are zero while  $E_{1r}$  and  $E_{1\theta}$  can be calculated based on (2).

##### B. Fields in the Plasma Layer—Region II ( $b \leq r \leq c$ )

Region II is a layer of uniform hot plasma with an ambient electron density of  $n_0$ , an electron thermal velocity of  $v_0$ , an electron plasma frequency of  $\omega_p$ , and an electron collision frequency of  $\nu$ . The basic equations for this region are that for the magnetic field  $H_2$  and the perturbed electron density  $n_1$ . They are as follows:

$$(\nabla^2 + k_e^2)H_2 = 0 \quad (3)$$

where

$$k_e^2 = \omega^2\mu_0\epsilon_0 \left[ \left( 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} \right) - j \frac{\omega_p^2\nu}{\omega(\omega^2 + \nu^2)} \right] \quad (4)$$